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MATHSCOUNT®

2023 Mock National Competition

Sprint Round problems 1-30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathslete. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ______________________ Date ____________

Printed Name ______________________

State ______________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wristwatch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to the lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Marks	Scorer's Initials

The ZeMC competition series is made possible by the contributions of the following problem-writers and test-solvers:

Anchovy, asbodke, bissue, Geometry285, iamhungry, ihatemath123, Jiseop55406, kante314, Olympushero, peace09, P_Groudon, and Significant.

1. £ ____________

A bakery charges £5 for one box of a dozen doughnuts. What is the smallest amount of money that Douglas must spend to buy at least 100 doughnuts?

2. ____________

Let $x * y = x^y$ and $x \circ y = y^x$. What is $(2 * -1) \circ (4 * 2)$?

3. ____________ kph

Ronald drove his lorry nine kilometers in twenty minutes. If he had driven the same distance in two fewer minutes, how much faster would his average speed have been?

4. ____________

If m and n are positive integers such that $m \div n = 1.68$, what is the minimum possible value of $m + n$?

5. £ ____________

Elizabeth, Mary and Victoria eat at a restaurant. To cover the bill, Elizabeth pays £20, Mary pays £23 and Victoria pays £24. Their payment receives £16 in change, which they distribute such that each friend ends up paying the same total amount. How much of the change does Elizabeth receive?

6. (,) Triangle ABC lies on the Cartesian plane with centroid G. The points G, A, B and C are located at $(1, 3)$, $(2, 5)$, $(-4, -3)$ and (x, y), respectively. What is the ordered pair (x, y)?

7. ______ stops A train stops at each station for exactly 1 minute, and takes exactly 3 minutes to get between consecutive stations. Victoria enters the train at the first station on the line, right before it departs, and rides for 75 minutes before getting off at the last stop on the line. How many stations are there on the line?

8. ______ Lily thinks of two distinct positive integers. She notices that neither of the integers is a perfect square, yet their product is a perfect square. What is the smallest possible sum of the two numbers Lily could be thinking of?

9. ______ days How many days are there in a year such that the numerical month is a divisor of the day of the month? (For example, the 20th of April is one day to count, since 4 is a divisor of 20.)

10. ______ units In $\triangle ABC$, $AB = 5$ and $BC = 6$. If the distance from C to $\overline{AB}$ is 4, what is the distance from A to $\overline{BC}$? Express your answer as a common fraction.

11. ________________

Jason rolls a fair six-sided die three times. What is the probability that the first roll is less than the second roll and the second roll is less than the third roll? Express your answer as a common fraction.

12. ________ questions

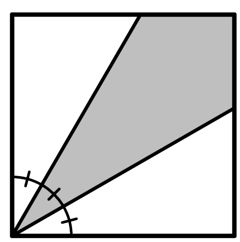
Evan took a 120-question test. Two thirds of the questions that Evan solved were among the first 60 questions, and three quarters of the questions that Evan didn't solve were among the last 60 questions. How many questions did Evan solve in total?

13. ________________

Define the *happiness* of a number to be the sum of its digits. What is the smallest number greater than 4,321 with *less* happiness than that of 4,321?

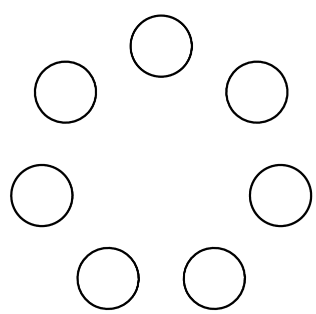
14. ________ units^2

The square below has a side length of 6 units. One of its right angles is trisected. What is the area of the shaded region? Express your answer in simplest radical form.



15. ________ ways

How many ways are there to arrange the numbers 1 through 7 in a circle, such that each number is besides at least one multiple or divisor of itself? (Rotations and reflections of an arrangement are considered the same.)



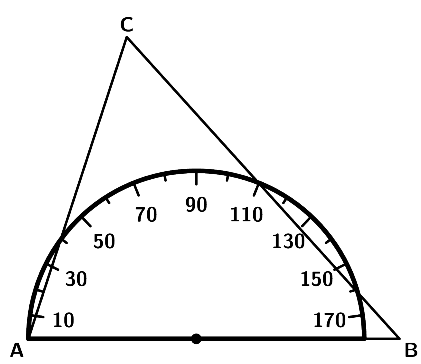
16. ________________ Define $d(x)$ over positive integers x as the number of divisors of x. What is the smallest positive integer n such that $\gcd(n, d(n)) = 70$?

17. ____________ units Two intersecting chords in a circle of area 9π are both 4 units long. If they intersect at a 60° angle, what is the distance between their midpoints? Express your answer in simplest radical form.

18. ________________ What is $\sqrt{401}$, rounded to the nearest thousandth?

19. ____________ digits The integer X has 14 digits when written in base 5, and the integer Y has 34 digits when written in binary. What is the maximum number of digits the value $X \times Y$ could contain, when expressed in base 10?

20. ____________ degrees A protractor is placed on top of a diagram of a triangle, as shown below. What is the measure of $\angle ACB$?



21. ________ permutations

How many permutations of the word "MATHSCOUNT" are there that contain at least one of the substrings "MOHS" or "STUC"?

22. ________

Ms. Maudsley writes down six real numbers. The median and mean of these numbers are 25 and 33, respectively. After Ms. Maudsley erases one of these numbers, the median and mean of the remaining five numbers are 22 and x, respectively. What is the maximum possible value of x?

23. ________ orders

A football match between two teams ended with a final score of 7 goals to 4 goals. How many different orders could these goals have been scored in, given that the team that won always had at least as many goals as the team that lost?

24. ________ numbers

Call an integer *doozy* if it is a multiple of 12 whose digits sum to 12. How many numbers *with nonzero digits* are doozy?

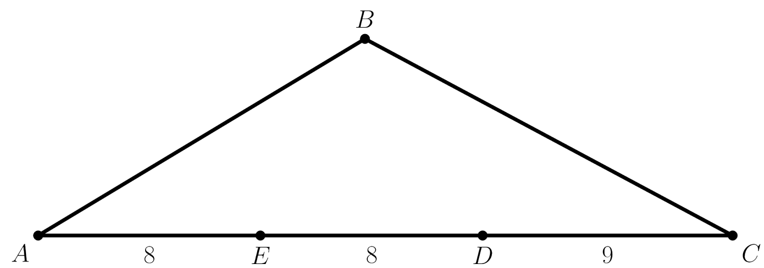
25. ________ ordered triples

How many ordered triples of positive integers (a, b, c) satisfy

$$\sqrt{a + \sqrt{b + \sqrt{c}}} = 5?$$

26. ________ units^2

In triangle ABC, $\angle ABC$ is obtuse. Let D be the point on $\overline{AC}$ such that $\angle DBA = 90^\circ$, and let E be the point on $\overline{AC}$ such that $\angle EBC = 90^\circ$. If $AE = 8$, $ED = 8$ and $DC = 9$, what is the area of $\triangle ABC$? Express your answer as a common fraction.



27. ________

Mr. Binnings picks two positive real numbers x and y, and writes down the following equations down:

$$\boxed{\begin{cases} xy + x + y &= 16 \\ xy + x^2 + y^2 &= 40 \\ xy + x^3 + y^3 &= \heartsuit \end{cases}}$$

What number represents the $\heartsuit$?

28. ________

Mathilda designs two new solids based off of a regular tetrahedron. Solid A is the unique solid whose vertices are the centroids of each face of the tetrahedron. Solid B is the unique solid whose vertices are the midpoints of each edge of the tetrahedron. What is the ratio of the volume of solid A to the volume of solid B? Express your answer as a common fraction.

29. ________

If x and y are integers such that

$$x^2 + 4xy + 5y^2 = 50,$$

what is the sum of all possible distinct values of xy?

30. ________

A parabola's vertex is at the origin of the Cartesian plane. It is tilted such that it passes through the points $(1, 0)$ and $(0, 2023)$. To the nearest integer, what is the slope of the parabola's axis of symmetry?