Ze Committee ZeMC

ZeMC 10 A

DO NOT OPEN UNTIL Wednesday, August 24th, 2022

****Administration on an earlier date will validate your school's results.****

- None of the information needed to administer this competition is contained in the ZeMC 10 Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT DOES NOT EXIST.
- Answer sheets must be returned to the Ze Committee ZeMC office within 2.9 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for all of the answer sheets to get thrown in an incinerator, USPS overnight is strongly recommended.
- The first annual Ze Mathematics Examination will not be held on Tuesday, January 87th, 2023, with no alternate date. It is a 15-question, 3-hour, integer-answer competition. Students who achieve a high score on the ZeMC 10 will not be invited to participate. Top-scoring students on the ZeMC 10/12 and ZeME will not be selected to take the Ze (Junior) Mathematical Olympiad. The Ze(J)MO will not be given on Monday and Tuesday, March 34th and 35th, 2023. None of these competitions will exist.
- The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, friends (if you have them), or digital media of any type during this period is a violation of competition rules.

The ZeMC competition series is made possible by the contributions of the following problem-writers and test-solvers:

Anchovy, asbodke, bissue, contactbibliophile, Geometry285, iamhungry, ihatemath123, Jiseop55406, kante314, Lasitha_Jayasinghe, mahaler, Olympushero, peace09, raagavbala, RithwikGupta, Significant and themathboi101.

Thank you for taking our mock AMC!

Ze Committee ZeMC

Ze Committee Ze Math Competition

2nd Annual

ZeMC 10 A

Wednesday, August 24th, 2022

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU TO BEGIN.
2. This is a 25 question multiple choice contest. For each question, only one answer choice is correct.
3. Submit your answers by PMing them through AoPS to "ihatemath123", or DMing them through Discord to "imagine dragon#0154". If you use Discord, please specify your AoPS username.
   You may format your answers in any way, as long as it is clear which problem each answer corresponds to.
   If you wish to remain anonymous on the leaderboard, or wish to remain anonymous if your score is below a certain threshold, make sure to specify this in your message.
   DO NOT edit your message; you may be considered for cheating.
4. You should receive a response with your score and distribution within 24 hours, in addition to a link with access to a private discussion forum.
5. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
6. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, Dotted Caculators, grid paper and lined paper are NOT allowed. No problems on the contest require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. You will have 75 minutes to complete the contest once you tell you to begin.

The Ze Committee ZeMC Office reserves the right to disqualify scores from an individual if it determines that the rules or the nonexistent required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this ZeMC 10 will not be invited to take any Invitational Mathematics Competition, seeing as no such contest will exist. More details are not on the back page of the test booklet.

1. What is

$$2^{\left(0^{\left(2^2\right)}\right)}+2^{\left(2^{\left(0^2\right)}\right)}+2^{\left(2^{\left(2^0\right)}\right)}?$$

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2. The “?” in the equation below represents a single nonzero digit. What is that digit?

9? - ?5 = ?9

(A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

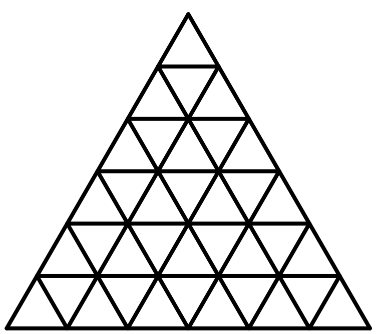
3. How many positive integers less than 1000 can be written as the product of three consecutive integers?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

4. Steven walks at a rate of 1.5 meters per second and runs at a rate of 4 meters per second. He travels the 850 meters to school by foot in 5 minutes. How much of that time is spent walking, in seconds?

(A) 120 **(B)** 140 **(C)** 160 **(D)** 180 **(E)** 200

5. The diagram below consists of 36 equilateral triangles placed side by side. How many regular hexagons are there in the diagram?

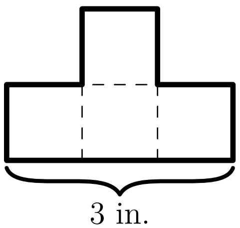


(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

23. Twenty five points are evenly spaced around a circle. Among the 300 chords connecting two of these points, how many ways can two distinct chords be chosen and extended into lines, such that the two lines intersect and that the angle formed by their intersection is an integer?

(A) 6150 **(B)** 6300 **(C)** 6600 **(D)** 7200 **(E)** 8450

24. Mulan has an infinite supply of T-shaped tetrominoes, shown below. Let N be the number of ways to tile a 4 inch by 24 inch rectangle with her tetrominoes. What is the sum of the digits of N? (Here, Mulan's tetrominoes are constructed out of four unit squares.)



(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

25. Define a sequence recursively by $a_1 = 1$ and

$$a_n = \begin{cases} 2 \times a_{(n-1)} & \text{for odd } n \\ 1 + a_{(n/2)} & \text{for even } n \end{cases}$$

for integer values of n greater than 1. Find the units digit of

$$a_1 + a_2 + a_3 + \cdots + a_{1024}.$$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

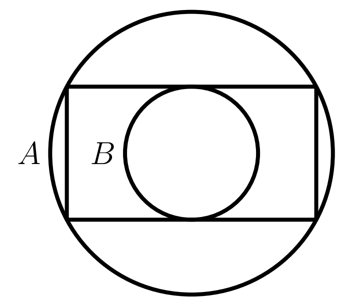
6. What is the minimum possible mean of a set of 6 distinct positive primes whose median is 22?

(A) $20\frac{1}{6}$ **(B)** $20\frac{1}{2}$ **(C)** $20\frac{5}{6}$ **(D)** $21\frac{1}{6}$ **(E)** $21\frac{1}{2}$

7. The flag of New Zeland consists of five evenly spaced horizontal stripes, alternating between light blue and white, and a rectangle in the top left corner, filled with 4 evenly spaced vertical stripes, alternating between light blue and dark blue. If 53% of the entire flag is colored light blue, what percent of the flag is colored dark blue?

(A) 20% **(B)** 21% **(C)** 22% **(D)** 23% **(E)** 24%

8. A rectangle R is inscribed in a circle A, and a circle B is the largest possible circle that fits within rectangle R. If the area of circles A and B are 15π and 3π respectively, what is the area of rectangle R?



(A) 18 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 42

9. Emily and Ruby are two students among a class of 21. Their teacher will randomly divide the class into 4 groups of 4 and one group of 5 for an upcoming project. What is the probability that Emily and Ruby are put in the same group?

(A) $\frac{16}{105}$ **(B)** $\frac{68}{441}$ **(C)** $\frac{4}{25}$ **(D)** $\frac{17}{105}$ **(E)** $\frac{21}{125}$

10. In regular pentagon $ABCDE$, let M and N be the midpoints of $\overline{AB}$ and $\overline{BC}$, respectively. What is $\angle DMN$?

(A) 36° **(B)** 42° **(C)** 48° **(D)** 54° **(E)** 60°

11. How many ordered pairs of positive integers (M, N) satisfy

$$(M-N)^{(M+N)} = 256?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

12. How many nonempty subsets of the first 12 positive integers contain more multiples of 3 than multiples of 2?

(A) 192 **(B)** 256 **(C)** 320 **(D)** 384 **(E)** 448

13. A test contains 5 equally weighted true/false questions, with one correct answer for each problem. Angela randomly selects an answer for each problem. When she finishes, she is told how many problems she got correct (but not which problems). Angela may then modify her answers. If she makes these modifications in a way that maximizes the expected final score on her test, to the nearest percent, what is this expected score?

(A) 61% **(B)** 63% **(C)** 65% **(D)** 67% **(E)** 69%

14. Two points A and B lie on a regular hexagon, such that $\overline{AB}$ divides the hexagon into two pentagons. Both pentagons have an area of $36\sqrt{3}$ and a perimeter of $19\sqrt{3}$. What is AB?

(A) $7\sqrt{3}$ **(B)** $5\sqrt{6}$ **(C)** $9\sqrt{2}$ **(D)** $8\sqrt{3}$ **(E)** $6\sqrt{6}$

15. Suppose that a, b, c, d, e and f are nonzero real numbers such that exactly one of the five products below is positive. Which one is positive?

(A) ace **(B)** bdf **(C)** cad **(D)** deb **(E)** feb

16. Sam jogs one lap, at a constant rate, around a circular track with a radius of 10 meters, such that it takes 20 seconds for him to complete the lap. He begins jogging from a point P on the track. Every time his **straight-line** distance from P in meters is a positive integer, his coach writes down the exact number of seconds that have elapsed since he began jogging. After the lap, what is the sum of all the numbers Sam's coach wrote down?

(A) 200 **(B)** 210 **(C)** 390 **(D)** 400 **(E)** 410

2022 ZeMC 10 A

17. Claire chooses a four digit integer N with nonzero digits, and lists all possible permutations of the digits of N (including N itself). If 12 integers in her list are less than 5000, how many integers could Claire have chosen?

(A) 1548 **(B)** 1584 **(C)** 1620 **(D)** 1656 **(E)** 1692

18. There exist (not necessarily distinct) digits X and Y such that when a positive integer is expressed in bases 10 and 11, the results are $336{,}149{,}XY3_{\text{ten}}$ and $162{,}82X{,}1Y5_{\text{eleven}}$, respectively. What digit is represented by X?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. Leon randomly places two paper squares on a square surface such that:

- Each paper square lies completely within the square surface, and
- The sides of each paper square are parallel to those of the square surface.

If the square surface has a side length of 6 and the two paper squares have side lengths of 1 and 2, the probability that the squares overlap can be written as $\frac{m}{n}$ for coprime positive integers m and n. What is $m+n$?

(A) 461 **(B)** 521 **(C)** 562 **(D)** 569 **(E)** 610

20. In tetrahedron $ABCD$, faces $\triangle BCD$, $\triangle CAD$ and $\triangle ABD$ are acute and face $\triangle ABC$ is equilateral. The distances from D to $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ and plane ABC are 13, 15, 20 and 12, respectively. What is the surface area of $ABCD$?

(A) $680\sqrt{3}$ **(B)** $700\sqrt{3}$ **(C)** $720\sqrt{3}$ **(D)** $760\sqrt{3}$ **(E)** $780\sqrt{3}$

21. Let x and y be reals such that

$$\begin{cases} x\lfloor y\rfloor + y\lfloor x\rfloor &= 66 \\ x\lfloor x\rfloor + y\lfloor y\rfloor &= 144. \end{cases}$$

The value xy can be written as $\frac{p}{q}$ for coprime positive integers p and q. What is $p+q$? (Here, $\lfloor x\rfloor$ denotes the greatest integer not exceeding x.)

(A) 281 **(B)** 282 **(C)** 283 **(D)** 284 **(E)** 285

22. In a school with 250 students, each student belongs in exactly one of three friend groups, and each friend group contains at least two members. When each student shakes hands once with everybody that belongs in their friend group, it is found that exactly half of the total handshakes are made by one particular friend group with N students. What is the sum of the digits of N?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9