Ze Committee ZeMC

ZeMC 10

DO NOT OPEN UNTIL Sunday, October 24th, 2021

Administration On An Earlier Date Will Precede Eventual Death

- None of the information needed to administer this competition is contained in the ZeMC 10 Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT DOES NOT EXIST.
- Answer sheets must be returned to the Ze Committee ZeMC office within 2.9 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for all of the answer sheets to get thrown in an incinerator, USPS overnight is strongly recommended.
- The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, friends (if you have them), or digital media of any type during this period is a violation of competition rules.

The ZeMC 10 is made possible by the contributions of the following problem-writers and test-solvers:

Geometry285
ihatemath123
Kante314
Ladka13
lopsmoss
mathboy282
sparklyflowers
x3yukari

Ze Committee ZeMC

Ze Committee Ze Math Competition

First Annual

ZeMC 10

Sunday, October 24th, 2021

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU TO BEGIN.
2. This is a 25 question multiple choice test. For each question, only one answer choice is correct.
3. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
4. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, Dotted Caculators, grid paper and lined paper are NOT allowed. No problems on the test require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. You will have 75 minutes to complete the test once you tell you to begin.
7. Note that this contest has already concluded - if you continue, you are taking this mock test unofficially. To find answers, solutions and other tests, visit Ze Committee's website, found at `https://ihatemath123.github.io/ZeMC/`.

The Ze Committee ZeMC Office reserves the right to disqualify scores from an individual if it determines that the rules or the nonexistent required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Usually this section of the booklet would be allocated for an italicized paragraph explaining how students that do exceptionally well on this test would get to take the AIME. However, since this mock AMC did not come with an AIME, there is nothing to write here. However, if there was not an italicized paragraph here, the test booklet would look quite ugly, so here is some random text anyways. From a distance, nobody would suspect that this italicized paragraph is not explaining how students that do exceptionally well on this test would get to take the AIME.

1. Find $\frac{21}{20} - \frac{20}{21}$.

   (A) $-\dfrac{841}{420}$ **(B)** $-\dfrac{41}{420}$ **(C)** $\dfrac{41}{420}$ **(D)** $\dfrac{841}{420}$ **(E)** 2

2. Given that Luke's teachers assign homework at the same constant rate, if 3 teachers can assign 120 homework assignments to Luke in 1 hour, how many homework assignments can 1 teacher assign to Luke in 15 minutes?

   (A) 10 **(B)** 12 **(C)** 15 **(D)** 120 **(E)** 600

3. A treatment has a 90 percent success rate of curing those with CUBIC-27. If three patients with CUBIC-27 are selected at random, what is the probability that exactly one of the three patients will be succesfully cured?

   (A) 0.9% **(B)** 2.7% **(C)** 8.1% **(D)** 72.9% **(E)** 99.9%

4. Ray does 1 problem on January 1st, 2021. The next day, he does 2 problems, and each day, he does one more problem than the day before. On what day will he do his 100th problem?

   (A) January 10th
   (B) January 11th
   (C) January 12th
   (D) January 13th
   (E) January 14th

5. Alex traveled two hours longer and three miles further than Beth, but averaged 2 mph slower. If the sum of their times spent travelling was 5 hours, what was the sum of the distances they traveled, in miles?

   (A) 4 **(B)** 7 **(C)** 10 **(D)** 14 **(E)** 18

6. When 2021! is written in base 20, how many zeroes are there at the end?

   (A) 202 **(B)** 503 **(C)** 504 **(D)** 509 **(E)** 2013

7. How many ordered pairs of positive integers (a, b) exist such that

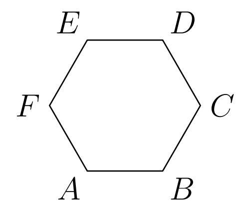
   $$\operatorname{lcm}(a, b) = 144?$$

   (A) 6 **(B)** 16 **(C)** 33 **(D)** 35 **(E)** 45

25. Call a number rotary if it is not a multiple of 10, and if each of its digits is either a 0, 1, 6, 8 or 9. Rotating a rotary number reverses the order of its digits; also, when rotating a rotary nubmer, all occurences of the digit 6 are replaced with 9's, and vice versa. For example, 8869 is a rotary number, and when rotated, it becomes 6988.
Merlin takes a rotary number n, rotates it, and multiplies the resulting number by 3. He then takes that number, rotates it again, and divides the result by 3. The resulting number is 11 digits long, and a multiple of 9. Find the number of possible values of n.

(A) 22 **(B)** 24 **(C)** 48 **(D)** 72 **(E)** 512

8. Points A, B, C, D, E, F, G and H are coplanar, such that $ABCDEF$ is a regular hexagon with an area of 2 square units; $ABGH$ is a square. If $x = [HEF]$, find the sum of the possible values of x.



(A) $\frac{2}{3}$ **(B)** 1 **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $\frac{4}{3}$ **(E)** $\sqrt{3}$

9. At Kostko, Circland is starting a new egg service. They package their eggs in a strange way: each box holds 15 eggs, with all of the eggs in a single row. There are seven brown eggs and eight white eggs in the box, and regulation states that no two brown eggs be next to each other in the box. If eggs of the same color are indistinguishable, how many orderings are there for the eggs in the box?

(A) 1 **(B)** 7 **(C)** 8 **(D)** 28 **(E)** 36

10. Victor has a four digit number N. He takes the last two digits of N and multiples the number they form by 101. He notices that the absolute value difference between his original number and his new number is 1000. Find the number of possible values for his original number.

(A) 140 **(B)** 155 **(C)** 170 **(D)** 185 **(E)** 200

11. If (x, y) is a point on the circle $x^2 + y^2 = 1$ and the distance from (x, y) to $(0, 1)$ is $\frac{5}{4}$, what is the value of y?

(A) $\frac{3}{8}$ **(B)** $\frac{7}{32}$ **(C)** $\frac{5}{16}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

12. Let x and y be real numbers such that

$$\begin{cases} |x| + x + y &= 8 \\ x + |y| - y &= 14. \end{cases}$$

Find the value of $|x| + |y|$.

(A) 4 **(B)** 6 **(C)** 10 **(D)** 22 **(E)** 30

13. Frédéric has 16 pieces on his playlist; piece i, for $1 \le i \le 16$, is $i+4$ minutes long. How many combinations of two distinct pieces can he pick such that the total length of the two pieces does not exceed 31 minutes? (The order of his pieces does not matter.)

(A) 64 **(B)** 100 **(C)** 110 **(D)** 115 **(E)** 120

14. If x is a real number between 0 and 10 inclusive, how many solutions does $x^2 = \lceil x \rceil \lfloor x \rfloor$ have?

(A) 10 **(B)** 11 **(C)** 19 **(D)** 20 **(E)** 21

15. Xi and Jinping each tell each other their favorite 2 digit positive integer. Xi notices that the tens digit of his number is the number of factors in Jinping's number, and that the units digit of Jinping's number is the number of factors in Xi's number. The largest possible value of Jinping's number is N. Given that Jinping's favorite number is N, find the sum of the digits of Xi's number.

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

16. Call a number a "nice palindrome" if it is both a multiple of 42 and a palindrome. Let the sum of the 5 smallest 5 digit nice palindromes be N. What is the sum of the digits of N?

(A) 3 **(B)** 6 **(C)** 15 **(D)** 18 **(E)** 27

17. An ellipse with the equation $x^2 + \frac{y^2}{4} = 1$ and a square are drawn. The square has vertices at $(-1,-1)$, $(-1,1)$, $(1,-1)$ and $(1,1)$. Let the area of the region inside the square but outside of the ellipse be A. $6A$ can be written as $a - b\sqrt{c} - d\pi$. Find $a+b+c+d$.

(A) 17 **(B)** 20 **(C)** 23 **(D)** 34 **(E)** 37

18. Let $S = \{1,2,3,4,\ldots,9,10\}$. A subset of S is called shiny if it is not empty, and if it contains more odd numbers than even numbers. How many shiny subests of S are there?

(A) 252 **(B)** 292 **(C)** 336 **(D)** 386 **(E)** 512

19. Triangle ABC is constructed such that $\angle B = 90°$ and $\angle A = 30°$. Let D be a point on $\overline{AB}$ between A and B such that $AD = 1$, and let E be a point on $\overline{AC}$ such that $ED = 13$ and $EB = 8$. What is the area of triangle ABC?

(A) $\frac{64\sqrt{3}}{3}$ **(B)** $32\sqrt{3}$ **(C)** $48\sqrt{3}$ **(D)** $\frac{128\sqrt{3}}{3}$ **(E)** $64\sqrt{3}$

2021 ZeMC 10

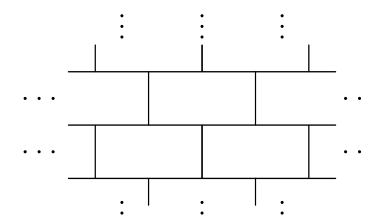
20. Remilia has an unfair coin that has a probability of $\frac{m}{n}$ to come up heads, where m and n are relatively prime positive integers. When Remilia flips the coin 26 times, the probability she gets exactly 3 heads is k times the probability she gets 1 heads, where k is a positive integer. Let N be the sum of all possible values of $n-m$. Find the remainder when N is divided by 5.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

21. Call a polynomial $P(x)$ fuzzy if: **a)** The roots of $P(x)$ are (not necessarily distinct) positive primes; **b)** The sum of the coefficients of $P(x)$ is exactly 24, and **c)** The leading coefficient of $P(x)$ is a positive integer. Let a be the absolute value of the constant term of $P(x)$, and b be the greatest odd divisor of a. Compute the number of distinct possible values of b over all fuzzy polynomials.

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

22. In the infinite brick wall below, every brick is a 2 by 4 inch rectangle, and each row is a duplicate of the previous row, shifted 2 inches to the right. A 3 inch by 3 inch square is randomly placed on the brick wall such that its sides are parallel to the sides of the bricks. Compute the expected number of bricks that are touched by the square.



(A) 4 **(B)** $\frac{33}{8}$ **(C)** $\frac{17}{4}$ **(D)** $\frac{35}{8}$ **(E)** $\frac{9}{2}$

23. Let ABC be a non-degenerate triangle, and let M be the midpoint of BC. AB, AM and AC are integers which form an increasing geoemtric sequence (in that order). If $AM = 420$, how many possible values are there for BC?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

24. Regular hexagon $ABCDEF$ has a side length of 2. Let P be a point such that the sum of its distances from $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{EF}$ and $\overleftrightarrow{AF}$ is less than $12\sqrt{3}$. Find the area of the locus of all possible points P.

(A) $48\sqrt{3}$ **(B)** $54\sqrt{3}$ **(C)** $64\sqrt{3}$ **(D)** $72\sqrt{3}$ **(E)** $75\sqrt{3}$